

FINAL REPORT

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5. NAME OF INSTITUTION: University of Virginia

6. AUTHOR OF REPORT: C. O. Horgan

7. LIST OF MANUSCRIPTS PUBLISHED

[1] Saint-Venant's principle for antiplane shear deformations of linear piezoelectric materials (A. Borrelli, C. O. Horgan and M. C. Patria), *SIAM J. on Applied Mathematics*, **62**, 2002, 2027-2044.

[2] Internally pressurized radially polarized piezoelectric cylinders (D. Galic and C. O. Horgan), *J. of Elasticity*, **66**, 2002, 257-272.

[3] The stress response of radially polarized rotating piezoelectric cylinders (D. Galic and C. O. Horgan), *J. of Applied Mechanics* , **70**, 2003, 426-435.

[4] A two-point boundary-value problem for the axial shear of hardening isotropic incompressible nonlinearly elastic materials (C. O. Horgan, G. Saccomandi and I. Sgura), *SIAM J. on Applied Mathematics*, **62**, 2002, 1712-1727.

[5] Finite thermoelasticity with limiting chain extensibility (C. O. Horgan and G. Saccomandi), *J. of the Mechanics and Physics of Solids*, **51**, 2003, 1127-1146.

[6] Helical shear for hardening generalized neo-Hookean materials (C. O. Horgan and G. Saccomandi), *Mathematics and Mechanics of Solids*, **8**, 2003, 539-559.

[7] A Saint-Venant principle for shear band localization (C. O. Horgan and W. E. Olmstead), *J. of Applied Mathematics and Physics (ZAMP)* , **54**, 2003, 807-814.

[8] End effects for pre-stressed and pre-polarized piezoelectric solids in anti-plane shear (A. Borrelli, C. O. Horgan and M. C. Patria), *J. of Applied Mathematics and Physics (ZAMP)*, **54**, 2003, 797-806.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED

C. O. Horgan, (P I); D. Galic (MS May 2002)

MS Thesis Title : Axisymmetric Problems for Radially Polarized Piezoelectric Cylinders

9. REPORT OF INVENTIONS (BY TITLE ONLY):

None

NASA Contact

(1) Continued close contact maintained with Dr. M. Nemeth, NASA.

(2) Extensive contact with Prof. Daniel Inman, Director, Center for Intelligent Material Systems and Structures, Virginia Tech on potential applications of the work to smart structures technology. Prof. Horgan presented a seminar at Virginia Tech that has led to sustained interaction between their research groups. The importance of end effects in smart structural damping problems e.g. layered beams and plates with viscoelastic, metallic and PZT components is currently under investigation and promises to have a major impact on NASA technology.

SUMMARY OF RESEARCH

STATEMENT OF THE PROBLEMS STUDIED AND RESULTS:

The research carried out here builds on our previous NASA supported research on the general topic of *edge effects and load diffusion* in composite structures. Further fundamental solid mechanics studies were carried out to provide a basis for assessing the complicated modeling necessary for the multi-functional large scale structures used by NASA. An understanding of the fundamental mechanisms of load diffusion in composite subcomponents is essential in developing primary composite structures. Some specific problems recently considered were those of end effects in smart materials and structures, study of the stress response of pressurized linear piezoelectric cylinders for both static and steady rotating configurations, an analysis of the effect of pre-stressing and pre-polarization on the decay of end effects in piezoelectric solids and investigation of constitutive models for hardening rubber-like materials.

Our goal in the study of load diffusion is the development of readily applicable results for the decay lengths in terms of non-dimensional material and geometric parameters. Analytical models of load diffusion behavior are extremely valuable in building an intuitive base for developing refined modeling strategies and assessing results from finite element analyses. The decay behavior of stresses and other field quantities provides a significant aid towards this process. The analysis is also amenable to parameter study with a large parameter space and should be useful in structural

tailoring studies. Special purpose analytical models of load diffusion behavior are extremely valuable in building an intuitive base for developing refined modeling strategies and in assessing results from general purpose finite element analyses. For example, a rational basis is needed in choosing where to use three-dimensional to two-dimensional transition finite elements in analyzing stiffened plates and shells. The decay behavior of stresses and other field quantities furnished by this research provides a significant aid towards this element transition issue. A priori knowledge of the extent of boundary-layers induced by edge effects is also useful in determination of the instrumentation location in structural verification tests or in material characterization tests.

We have made considerable recent progress in investigation of end effects in smart materials and structures. In our first study [1], we have considered anti-plane shear deformations of *piezoelectric* solids and have examined the effect of electric/mechanical coupling on the decay of end effects. For some classes of piezoceramics (e.g. PZT-5H) the decay length for anti-plane shear on a semi-infinite strip coincides with that for the purely mechanical result for isotropic anti-plane shear i.e. *approximately one strip width*. For other symmetry classes, it is shown that *the decay length is much larger*. Thus, just as for the case of highly anisotropic materials previously investigated by the P.I., *slow diffusion* of end effects in piezoceramics can take place. Another issue investigated [8] is that of the influence of *pre-stress and pre-polarization* on the rate of stress decay. For materials like PZT-5H under pre-stress, it is shown in [8] that mechanical end effects decay at a rate slower than electrical end effects and that the stress decay rate is monotone decreasing as the amount of pre-stress (tensile or compressive) increases. Thus, *for large pre-stress*, we again obtain *a very slow rate of diffusion resulting in a decay length much larger than the strip width*.

Two other investigations of basic problems for piezoelectric solids have been carried out in [2] and [3]. In [2] we have examined the fundamental problem of a piezoelectric hollow cylinder, with inner radius a and outer radius b , subjected to internal pressure (see Figure 1). The material is radially polarized and has circumferentially orthotropic or radially orthotropic mechanical anisotropy (the anisotropy typical of carbon fibers). Two specific piezoceramics are examined in detail, namely PZT-4 and BaTiO₃. Three different sets of boundary conditions are considered. In Case 1, the cylinder is subjected to an internal uniform pressure, zero electric potential difference across the cylindrical annulus and traction-free boundary conditions on the outer surface. The tube acts as a *sensor* in this case. The results are shown in Figure 2, where the three curves for each material correspond to three different aspect ratios, namely a thin shell ($b/a = 1.3$), a hollow cylinder ($b/a = 2$) and a thick tube ($b/a = 4$). The top subfigure of Figure 2 depicts the induced electrical effect. The compressive radial stresses shown in the middle subfigure are monotone increasing while the tensile hoop stresses shown in the lower subfigure decrease from the inner to the outer surface. *These stress profiles are very similar to those of the purely mechanical problem*. These are shown in Figure 3 for an *elastic* cylinder with *elastic* constants identical to those of PZT-4 and BaTiO₃. In Case 2, both inner and outer surfaces are traction-free and a uniform potential difference is prescribed across the annulus (e.g. by placing electrodes on the inner and outer surfaces). Now the tube acts as an *actuator*. The results are shown in Figure 4. The induced radial compressive stress shown in the middle subfigure is no longer monotone but has an *interior* minimum. *The hoop stresses shown in the lower subfigure of Figure 4 are striking*. For each aspect ratio of the tube, the hoop stresses change from being *compressive* in the inner region to *tensile* in the outer region. As far as the P.I. is aware, such a sign change does *not* occur in purely mechanical problems. The ramifications of this sign change are seen in Case 3, which is a superposition of

Cases 1 and 2. The results are shown in Figure 5. Observe that in the lower subfigure of Figure 5 *that there is virtually no variation in the magnitude of the hoop stress throughout the cylinder*. For all three aspect ratios, the curves have flattened out. For a thick tube, the hoop stress is close to zero. *The implication of these results for failure are dramatic*. Internally pressurized elastic cylinders fail at the point of highest hoop stress which for orthotropic materials typically occurs at the inner surface. Here we have shown that for a piezoelectric cylinder, the hoop stress can be made uniform by applying a suitable electric field. Furthermore, the *radial stress* may now be the critical stress for failure as can happen, for example, in thermal loading of fibrous composites. We believe that these results have significant potential impact on NASA technology.

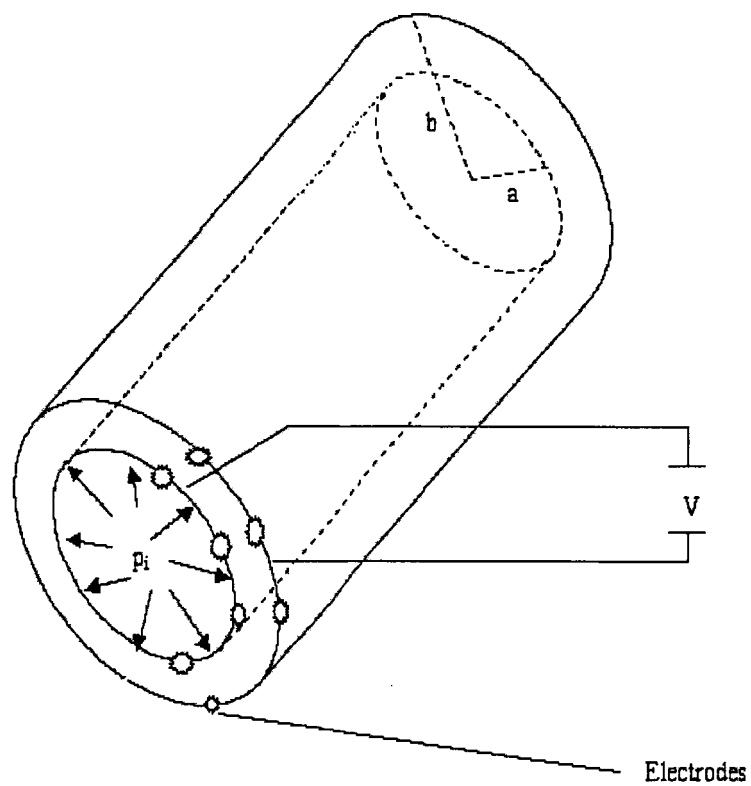
In [3], we consider the steady rotation about its axis of a piezoelectric cylinder under the same combination of loading as described above. The effect of rotation on the stress profiles is examined. For the special case of a uniformly rotating solid piezoelectric cylinder with traction-free surface and no applied electric charge, explicit closed-form solutions are obtained. The results are again strikingly different from the analogous purely mechanical problem.

Some problems in nonlinear elasticity have been investigated in [4-6]. The concern is with incompressible rubber-like materials that *harden at large strains*. Classical constitutive models for rubber, such as the Mooney-Rivlin or neo-Hookean model, do not capture this effect. We have considered two types of constitutive models that reflect material hardening, namely power-law models and a model that incorporates *limiting chain extensibility* at the molecular level. The latter model is mathematically tractable while capturing the essential physics. In [4] axial shear of a hollow tube bonded to a rigid hub is investigated analytically and numerically. Interesting boundary-layer and interior localization phenomena are demonstrated. In [5] the basic mechanical model is extended to nonlinear thermo-elasticity and a thermo-elastic axial shear problem is solved. The thermal response of rubber seals is of obvious importance to NASA. We plan to investigate the utility of our new constitutive model in more elaborate thermo-elastic settings in future work. In [7] we have shown that the temperature field in *shear band formation* in thermoviscoplastic solids is highly *localized*. Methods developed during our investigations of Saint-Venant's principle for nonlinear problems have surprisingly also turned out to be useful in the study of shear band localization.

Statement of the Impact of the Proposed Research

The research program carried out here should contribute to NASA's Structures and Materials Competency in several ways. We continue to show from specific problems, simple enough to be amenable to considerable analysis yet elaborate enough to be of practical significance, that end and edge effects in anisotropic laminated materials and smart structures can be far more severe than in homogeneous isotropic structures of the same geometry and under the same loads. A fundamental problem such as the pressurized cylinder problem has given rise to surprising new discoveries when material inhomogeneities or piezoelectric effects are taken into account. We believe that well-focussed fundamental research of the type carried out here has an important role to play in the advancement of NASA technology.

STRESS RESPONSE OF AN INTERNALLY PRESSURIZED PIEZOELECTRIC HOLLOW CYLINDER



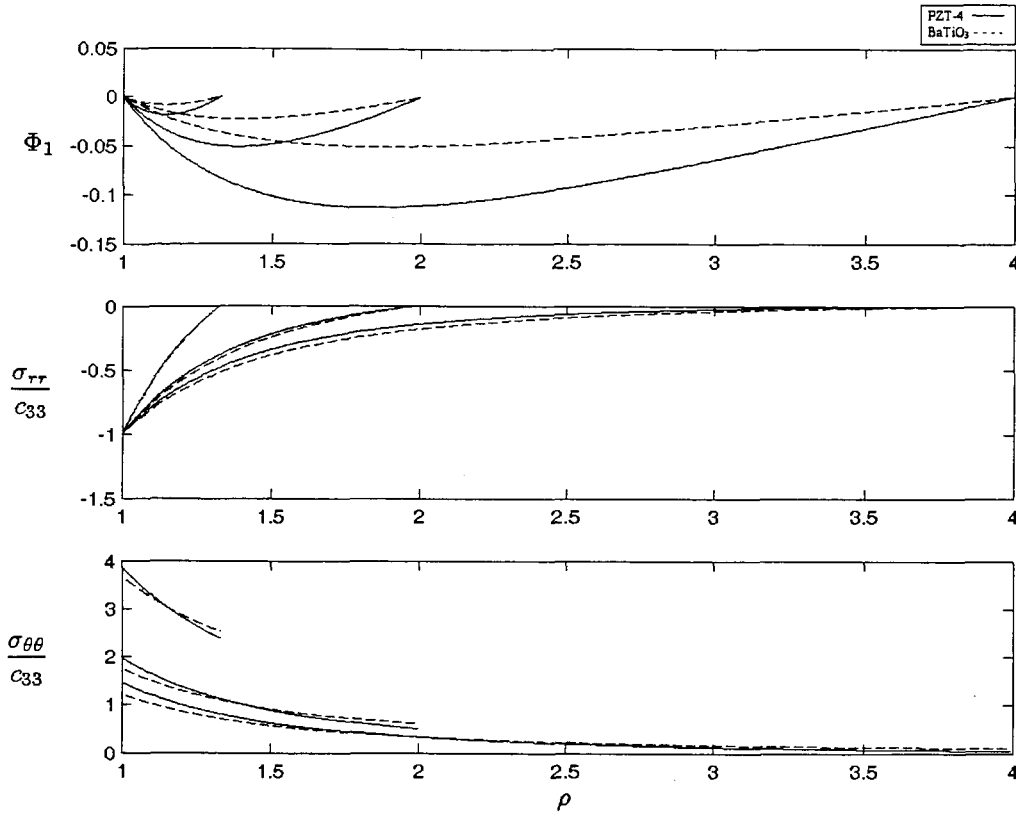


FIG. 2: CASE 1(SENSOR): UNIT APPLIED INTERNAL PRESSURE, NO APPLIED VOLTAGE

PLOTS OF INDUCED POTENTIAL, RADIAL AND HOOP STRESSES VERSUS r/a FOR CYLINDER ASPECT RATIOS $b/a = 1.3$ (THIN SHELL), $b/a = 2$ (HOLLOW CYLINDER), $b/a = 4$ (THICK TUBE)

C_{33} is the Young's Modulus in the radial direction (115×10^9 Pa for PZT-4, 146×10^9 Pa for BaTiO₃)

- **STRESS DISTRIBUTION IS SIMILAR TO PURELY ELASTIC PROBLEM**

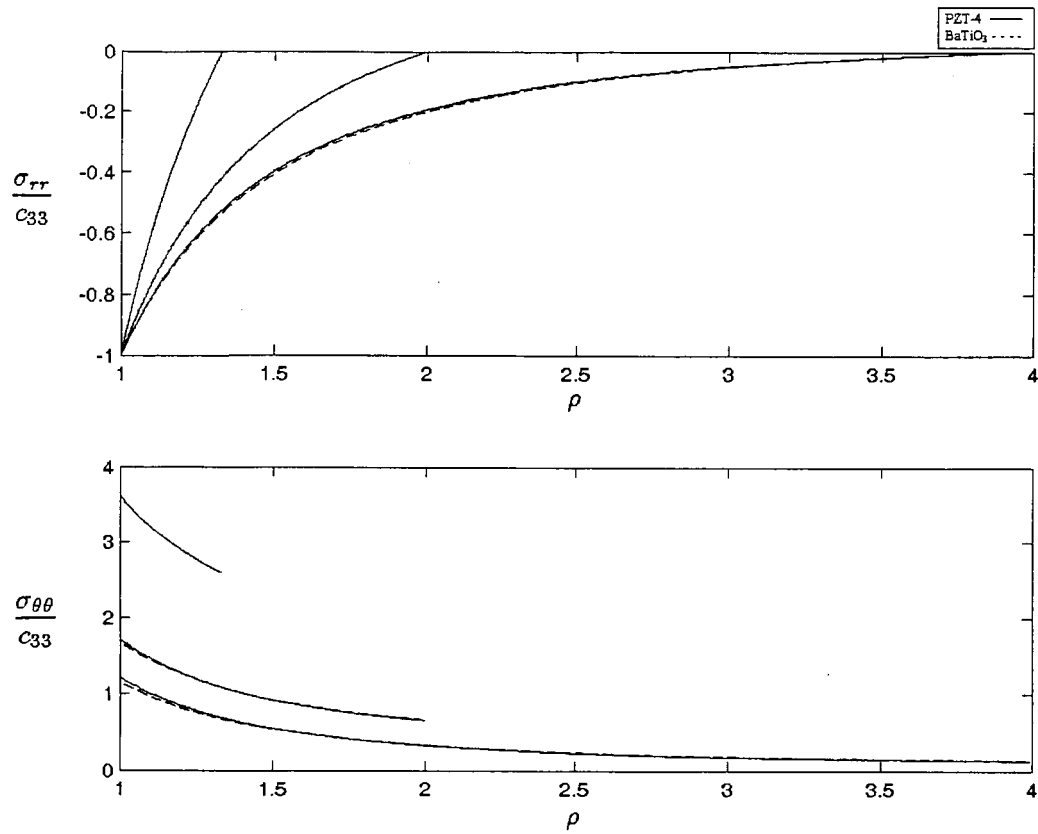


FIG. 3: PURELY ELASTIC CYLINDER WITH ELASTIC CONSTANTS IDENTICAL TO THOSE OF PZT-4 AND BaTiO₃ UNDER UNIT INTERNAL PRESSURE. CYLINDER ASPECT RATIOS $b/a = 1.3$ (THIN SHELL), $b/a = 2$ (HOLLOW CYLINDER), $b/a = 4$ (THICK TUBE).

C_{33} is the Young's Modulus in the radial direction (115×10^9 Pa for PZT-4, 146×10^9 Pa for BaTiO₃)

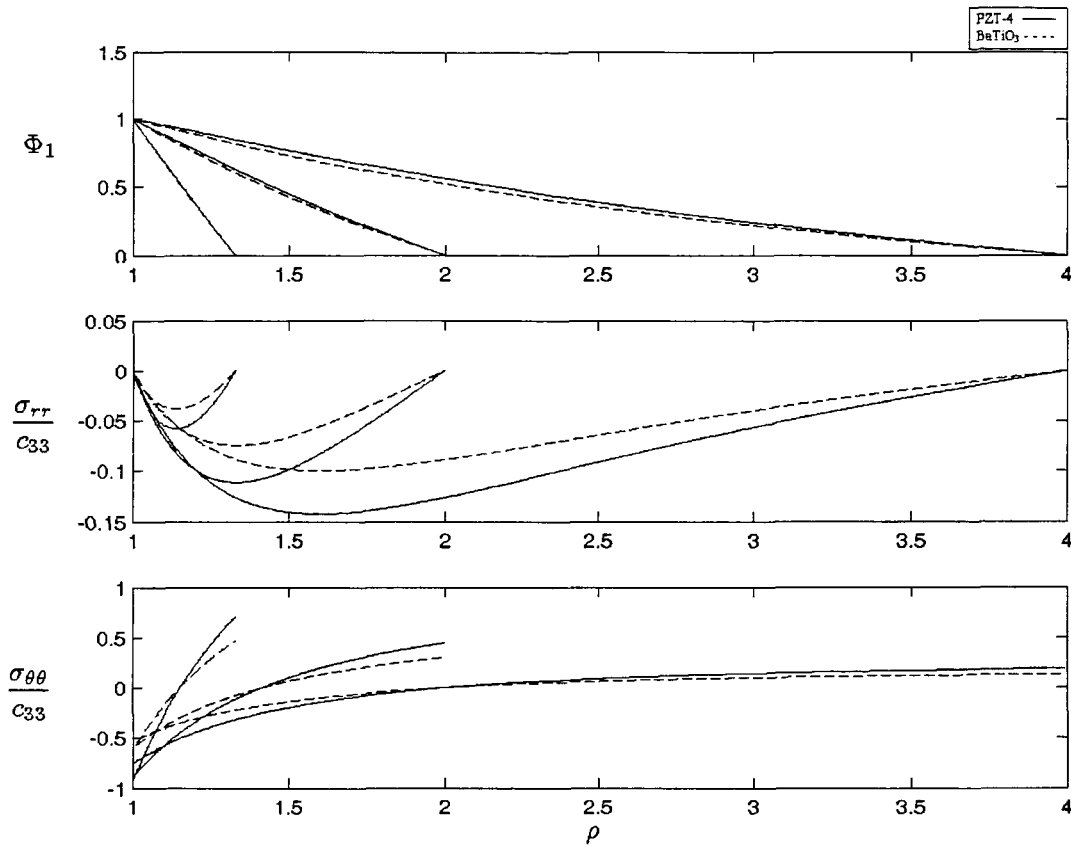


FIG. 4: CASE 2 (ACTUATOR): UNIT POTENTIAL DIFFERENCE, NO APPLIED TRACTIONS

PLOTS OF POTENTIAL, INDUCED RADIAL AND HOOP STRESSES VERSUS r/a FOR CYLINDER ASPECT RATIOS $b/a = 1.3$ (THIN SHELL), $b/a = 2$ (HOLLOW CYLINDER), $b/a = 4$ (THICK TUBE)

C_{33} is the Young's Modulus in the radial direction (115×10^9 Pa for PZT-4, 146×10^9 Pa for BaTiO₃)

- INDUCED COMPRESSIVE RADIAL STRESS HAS INTERNAL MINIMUM
- INDUCED HOOP STRESS CHANGES FROM COMPRESSIVE IN INSIDE REGION TO TENSILE IN OUTER REGION

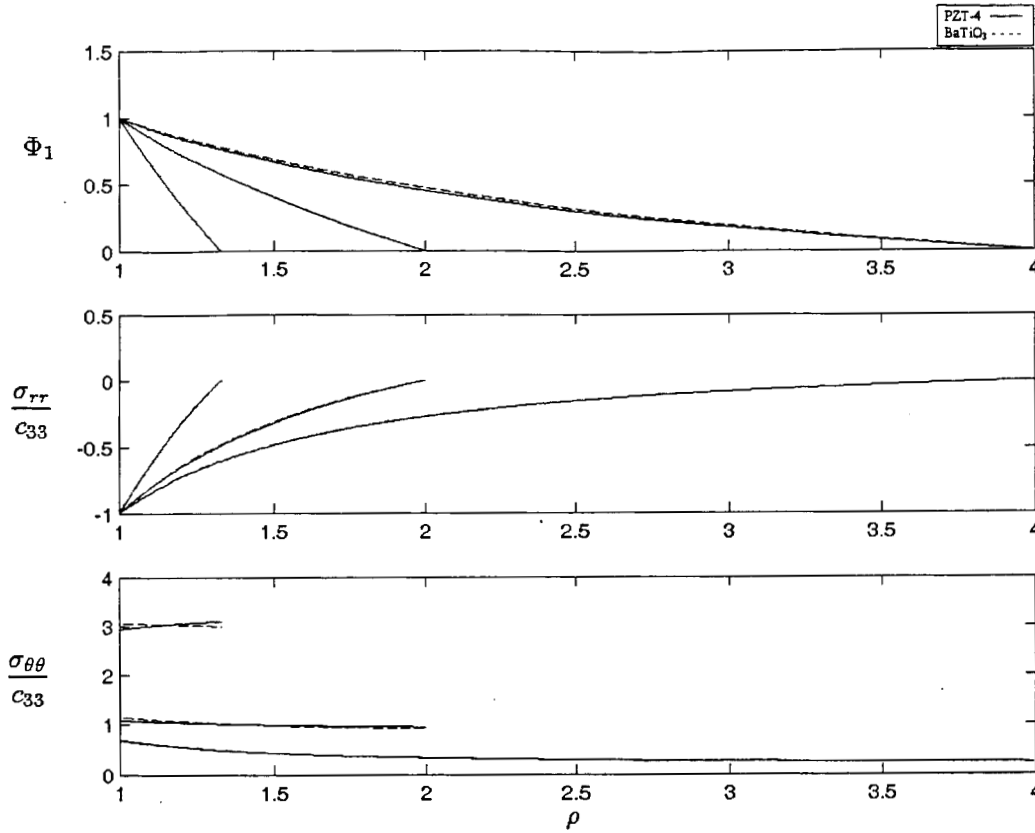


FIG. 5: CASE 3 (COMBINED LOADING): UNIT POTENTIAL DIFFERENCE, UNIT APPLIED INTERNAL PRESSURE

PLOTS OF POTENTIAL, RADIAL AND HOOP STRESSES
VERSUS r/a FOR CYLINDER ASPECT RATIOS $b/a = 1.3$ (THIN SHELL),
 $b/a = 2$ (HOLLOW CYLINDER), $b/a = 4$ (THICK TUBE)

C_{33} is the Young's Modulus in the radial direction (115×10^9 Pa for PZT-4,
 146×10^9 Pa for BaTiO₃)

- **RADIAL STRESS IS COMPRESSIVE AND MAGNITUDE IS MONOTONE DECREASING AS IN PURELY ELASTIC CASE**
- **HOOP STRESS IS VIRTUALLY UNIFORM ACROSS THE TUBE**
- **APPLICATION OF EXTERNAL ELECTRIC FIELD TO INTERNALLY PRESSURIZED CYLINDER NEUTRALIZES HOOP STRESS VARIATION THROUGH THE THICKNESS**